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Research Article

RADIO NUMBER OF CERTAIN GRAPHS WITH EXTENDED WHEELS

Kins Yenoke

PG & Research Department of Mathematics, Loyola college, Chennai 600 034, India

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ABSTRACT

A *radio labelling* of a connected graph G is an one-to-one mapping $f: V(G) \to N$ such that $d(u,w) + |f(u) - f(w)| \geq 1 + diam(G) \forall u, w \in V(G)$. The *radio number of* f denoted $rn(f)$, is the maximum number assigned to any vertex of G. The *radio number of* G, denoted $rn(G)$, is the minimum value of $rn(f)$ taken over all labelings $f$of G. In this paper we have defined and completely determined the radio number of certain graphs with extended wheels namely$WW(r)$, $SW(r)$, $KDW(r)$ and $HW(r)$.

INTRODUCTION

In the end of the twentieth century there is a need of assigning the maximum number of channels to FM radio stations within the fixed frequency band. The FM broadcast band used for FM broadcast radio by radio stations differs between different parts of the world. But the FM stations in most of the countries are assigned center frequencies at 200 kHz separation starting at 88.1 MHz and finished at 108MHz for a maximum of 100 stations. Broadcast regulators frequently manage the broadcast spectrum in order to minimize adjacent-channel interference. The main parameter for the interference is the distance between the transmitters. The problem of frequency assignment arises when different radio transmitters which operate in the geographical area interfere with each other when assigned to the same or closely related frequency channels. To avoid such interferences among the radio transmitters, Chartrand *et al*. [4] in 2001 introduced theradio labelling of graphs. The formal graph theoretical definition for radio labelling problem is stated as below.

A ***radio labelling*** of a connected graph G is an injection f from the vertices of G to the natural numbers such that $d(u,v) + |f(u) - f(v)| \geq 1 + diam(G))$ for every two distinct vertices u and v of G. The ***radio number of f***, denoted by $rn(f)$, is the maximum number assigned to any vertex of G. The ***radio number of G***, denoted by $rn(G)$, is the minimum value of $rn(f)$ taken over all labelings f of G.In the intervening years few more graph labelling's related to radio labelling's were introduced by different researchers.

Corresponding author:* **Kins Yenoke
PG & Research Department of Mathematics, Loyola college, Chennai 600 034, India

Among this, few important problems are antipodal radio labelling [5], radio mean labelling [8], radio mean square labelling [9] and antipodal radio mean labelling [1]. Determining radio number of a general graphs with diameter 2 is NP-hard [6], most of the results focus on particular classes of graphs and certain interconnection networks.

An Overview of the Paper

In the recent years a greater number of research articles are published in the radio assignment problems. The radio numbers for different families of graphs have been studied by several authors. Liu et.al [7]obtained the radio number for square cycles.RomanCada *et al*. [10]have determined the radio number of distance graphs. Bharati *et al*. [3] obtained the bounds for the radio number hexagonal mesh and lower bound for any connected simple graphs in terms of eccentricities. uniform theta graphs. Benson et.al [2] determined the radio numbers of all graphs of order n and diameter $n-2$.Vaidya *et al*.[11] studied the radio labelling of some cycle related graphs. In this paper we have define and completely investigated the radio number of certain graphs with extended wheels.

Lower and upper bounds

Bharati *et al*. [3] have proved the following result which we will use to find the lower bound for the graphs with extended wheels.

Theorem 3.1: (As Theorem 2 in[3])**:** Let G be a simple connected graph of order n. Let $n_0, n_1 \dots n_k$ be the number of vertices having eccentricities $e_0, e_1 \dots e_k$,where $diam(G) = e_0 > e_1 > \cdots > e_k = rad(G)$.Then $rn(G) \geq$

$$\begin{cases} n-2(d-e_k)+\sum_{i=1}^{k}2(d-e_i)n_i, & if\ n_k>1 \\ n-(d-e_k)-(d-e_{k-1})+\sum_{i=1}^{k}2(d-e_i)n_i, & if\ n_k=1 \end{cases}$$

The graph $WW(r)$

***Definition* 3.1:**Let $W_{n+1}=C_n+K_1$ be a wheel with $V(C_n)=\{w_i: 1\le i\le n\}$ and hub x. Let $W_{r+1}^i=C_r^i+K_1$be wheels with hubs $w_i, 1\le i\le n$, respectively. The graphs obtained is denoted by $WW(r), n>7$.The number of vertices in $WW(r)$ is $n(r+1)+1$ and the number of edges is $2n(r+1)$. The diameter of $WW(r)$ is 4.

***Theorem* 3.2:**The radio number of WW(r) is given by $rn(WW(r)=n(r+3), n>7$.

***Proof*:** Let $V(C_r^i)=\{v_{r(i-1)+j}: j=1,2\dots r\}, i=1,2\dots n$. We assume that the vertices v_i and w_i appear in the clockwise sense. We consider two cases.

***Case* 1:**n is even

Define a mapping $f: V(WW(r))\to N$ as follows

$f(v_{2(i-1)r+j})=\frac{n}{2}(j-1)+i+3, i=1,2\dots\frac{n}{2}, j=1,2\dots r.$

$f(v_{(2i-1)r+j})=\frac{n}{2}(r+j-1)+i+3, i=1,2\dots\frac{n}{2}, j=1,2\dots r.$

$f(w_{2i})=nr+3i+2, i=1,2\dots\frac{n}{2}.$

$f(w_{2i+1})=n\left(r+\frac{3}{2}\right)+3i+2, i=1,2\dots\frac{n}{2}-1.$

$f(w_1)=n(r+3)+2,\ f(x)=1$. See Figure 1.

We claim that f is a valid radio labelling.

Let $u,w\in V(WW(r))$.

***Case* 1.1**: u and w are any two vertices in the same cycle C_r^i.

***Case* 1.1.1:** If $u=v_{2(i-1)r+k}$ and $w=v_{2(i-1)r+l}$, then $f(u)=\frac{n}{2}(k-1)+i+3,\ f(w)=\frac{n}{2}(l-1)+i+3, 1\le k\ne l\le r$ and $d(u,w)\ge 1$. Since $n>7$ and $k\ne l$, we have $d(u,w)+|f(u)-f(w)|\ge 1+\left|\frac{n}{2}(k-l)\right|\ge 5$.

***Case*1.1.2:** If $u=v_{(2i-1)r+k}$ and $w=v_{(2i-1)r+l}$, then $f(u)=\frac{n}{2}(r+k-1)+i+3,\ f(w)=\frac{n}{2}(r+l-1)+i+3, 1\le k\ne l\le r$ and $d(u,w)\ge 1$. Therefore$d(u,w)+|f(u)-f(w)|\ge 1+\left|\frac{n}{2}(k-l)\right|\ge 5$, Since $n>7$ and $k\ne l$.

***Case* 1.2:**u and w are vertices in different cycles; $u\in C_r^k, w\in C_r^m, k\ne m$.

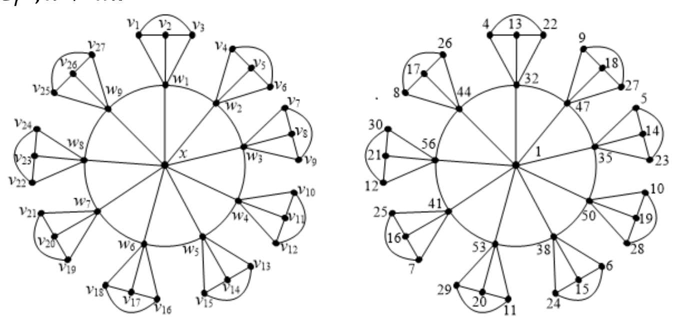

Figure 1 A graph WW(3) with n = 9 and a radio labelling

***Case* 1.2.1:** If $u=v_{2(k-1)r+l}$ and $w=v_{2(m-1)r+s}$, then $f(u)=\frac{n}{2}(l-1)+k+3,\ f(w)=\frac{n}{2}(s-1)+m+3, 1\le l,s\le r,\ 1\le k\ne m\le\frac{n}{2}$ and $d(u,w)=4$. Therefore $d(u,w)+|f(u)-f(w)|\ge 4+\left|\frac{n}{2}(l-s)+(k-m)\right|$.

If $l=s$, then $d(u,w)+|f(u)-f(w)|\ge 4+|k-m|\ge 5$, since $k\ne m$.

If $l\ne s$, then without loss of generality we may assume $l>s$. Therefore $d(u,w)+|f(u)-f(w)|\ge 4+\left|\frac{n}{2}(2-1)+\left(1-\frac{n}{2}\right)\right|=5$, since $k\ge 1$and $m\le\frac{n}{2}$.

***Case* 1.2.2:** If $u=v_{(2k-1)r+l}$ and $w=v_{(2m-1)r+s}$, then $f(u)=\frac{n}{2}(r+l-1)+k+3,\ f(w)=\frac{n}{2}(r+s-1)+m+3, 1\le l,s\le r,\ 1\le k\ne m\le\frac{n}{2}$ and $d(u,w)=4$. Therefore $d(u,w)+|f(u)-f(w)|\ge 4+\left|\frac{n}{2}(l-s)+(k-m)\right|$. For both the possibilities $l=s$ and $l\ne s$, we will arrive the same result as in case 1.2.1.

***Case* 1.2.3:**$u=v_{(2k-1)r+l}$ and $w=v_{2(m-1)r+s}, 1\le l,s\le r, 1\le k<m\le\frac{n}{2}$.

If $k>1$, then $f(u)=\frac{n}{2}(r+l-1)+k+3,\ f(w)=\frac{n}{2}(s-1)+m+3, 1\le l,s\le r,\ 2\le k<m\le\frac{n}{2}$ and $d(u,w)\ge 3$. Therefore $d(u,w)+|f(u)-f(w)|\ge 5$, since $s\le r, m$ and $l\ge 1, k\ge 2$.

If $k=1$ and $m=\frac{n}{2}$, then $d(u,w)=4$ and hence $d(u,w)+|f(u)-f(w)|\ge 4+1$, since $s\le r$and $l\ge 1$.

Otherwise, that is, if $k=1$ and $m<\frac{n}{2}$, then $d(u,w)\ge 3$. Also $|f(u)-f(w)|\ge 2$, since $s\le r$and $l\ge 1$.

***Case*1.3:**$u=v_i, 1\le i\le nr$ and $w=w_j,\ 1\le j\le n$

***Case* 1.3.1:** If $u=v_{2(k-1)r+l}$ and $w=w_{2s}$, then $f(u)=\frac{n}{2}(l-1)+k+3,\ f(w)=nr+3s+2, 1\le l\le r,\ 1\le k,s\le\frac{n}{2}$ and $d(u,w)\ge 2$. Since $l\ge 1, k\le\frac{n}{2}$ and $s\ge 1$, we have $d(u,w)+|f(u)-f(w)|\ge 2+\left|(nr+3s+2)-(\frac{n}{2}(l-1)+k+3)\right|\ge 2+\frac{n}{2}r+2>5$, since $n>7$.

***Case* 1.3.2:** If $u=v_{2(k-1)r+l}$ and $w=w_{2s+1}, 1\le l\le r,\ 1\le k\le\frac{n}{2},\ 1\le s\le\frac{n}{2}-1$, then $d(u,w)\ge 1$ and $|f(u)-f(w)|\ge\left|\left(n\left(r+\frac{3}{2}\right)+3s+2\right)-(\frac{n}{2}(l-1)+k+3)\right|$. Since $l\le r, k\le\frac{n}{2}$ and $s\ge 1$, we have $d(u,w)+|f(u)-f(w)|\ge 1+\left|\left(\frac{n}{2}r+\frac{3n}{2}+2\right)\right|>5$, Since $n>7$.

***Case* 1.3.3:**$u=v_{(2k-1)r+l}$ and $w=w_{2s}, 1\le l\le r,\ 1\le k\le\frac{n}{2},\ 1\le s\le\frac{n}{2}$.

If $2\le s\le\frac{n}{2}$, then $f(u)=\frac{n}{2}(r+l-1)+k+3,\ f(w)=nr+3s+2$ and $d(u,w)\ge 1$. Hence $d(u,w)+|f(u)-f(w)|\ge 1+\left|nr+3s+2-(\frac{n}{2}(r+l-1)+k+3)\right|\ge 5$, since $l\le r, k\le\frac{n}{2}$ and $s\ge 2$.

If $s=1$ with $l=r, k=\frac{n}{2}$ then $d(u,w)=3$ and $|f(u)-f(w)|=2$. Therefore $d(u,w)+|f(u)-f(w)|\ge 5$.

Otherwise, $d(u,w) \geq 1$ and $|f(u)-f(w)| \geq \left|\frac{n}{2}+2\right|$, since $l \leq r, k \leq \frac{n}{2}$. Hence $d(u,w)+|f(u)-f(w)| \geq 5$, since $n > 7$.

***Case* 1.3.4:** If $u = v_{(2k-1)r+l}$ and $w = w_{2s+1}$, $1 \leq l \leq r$, $1 \leq k \leq \frac{n}{2}$, $1 \leq s \leq \frac{n}{2}-1$, then $|f(u)-f(w)| = \left|\left(n\left(r+\frac{3}{2}\right)+3s+2\right)-\left(\frac{n}{2}(r+l-1)+k+3\right)\right| \geq \left|\frac{3n}{2}+2\right|$ and $d(u,w) \geq 2$. Since $l \geq r, k \geq \frac{n}{2}$, we have $d(u,w)+|f(u)-f(w)| \geq 5$.

***Case* 1.4:** If u is any vertex on the cycle and $w = x$, then $|f(u)-f(w)| \geq 3$ and the distance between them is exactly 2. Therefore $d(u,w)+|f(u)-f(w)| \geq 5$.

***Case* 1.5:** If $u = w_i$ and $w = x, 1 \leq i \leq n$, then $d(u,w) = 1$ and $|f(u)-f(w)| \geq nr+4 \geq 5$, since $n > 7$.

***Case* 1.6:** Suppose $u = w_{2k-1}$ and $w = w_{2l-1}$ or $u = w_{2k}$ and $w = w_{2l}$, $1 \leq k \neq s \leq \frac{n}{2}$, then $d(u,w) = 2$ and $|f(u)-f(w)| \geq 3$. Therefore $d(u,w)+|f(u)-f(w)| \geq 5$.

***Case* 1:7:** Suppose $u = w_{2k}$ and $w = w_{2l-1}$, $1 \leq k, l \leq \frac{n}{2}$, then $d(u,w) \geq 1$ and $|f(u)-f(w)| \geq \frac{3n}{2}-3$. Therefore $d(u,w)+|f(u)-f(w)| \geq 5$, since $n > 7$.

Hence $d(u,w)+|f(u)-f(w)| \geq 5$ for all $u,w \in V(WW(r))$.

Therefore, $rn(WW(r) \leq n(r+3), n > 7$. (1)

Next, we claim that f induces the minimum radio labeling.

We can easily see that, there are nr vertices with eccentricity $e_0 = 4$, n vertices with eccentricity $e_1 = 3$ and a single vertex with eccentricity $e_2 = 2$. That is, $n_0 = nr$, $n_1 = n$ and $n_2 = 1$. Hence by Theorem 3.1, $rn(WW(r) \geq n(r+1)+1+2n_1+4n_2-(d-e_2)-(d-e_1) = n(r+1)+1+2n+4-2 = nr+3n+2$.

That is, $rn(WW(r) \geq n(r+3), n > 7$. (2)

From (1) and (2), $rn(WW(r) = n(r+3), n > 7$

***Case* 2:** n odd

Define a mapping $f: V(WW(r)) \rightarrow N$ as follows:

$$f\left(v_{2(i-1)r+j}\right) = n(j-1)+i+3,\ j = 1,2\ldots r,\ i = 1,2\ldots\left\lceil\frac{n}{2}\right\rceil$$

$$f\left(v_{(2i-1)r+j}\right) = \left\lceil\frac{n}{2}\right\rceil+n(j-1)+i+3, j = 1,2\ldots r, i = 1,2\ldots\left\lfloor\frac{n}{2}\right\rfloor$$

$$f(w_{2i-1}) = nr+3i+2, i = 1,2\ldots\left\lceil\frac{n}{2}\right\rceil$$

$$f(w_{2i}) = nr+3\left\lceil\frac{n}{2}\right\rceil+3i+2, i = 1,2..\left\lfloor\frac{n}{2}\right\rfloor, f(x) = 1.$$

The rest of the proof is similar that of case 1.

The graph SW(r)

***Definition* 3.2:** Let $w_i, 1 \leq i \leq n$ be the vertices of a star graph S_{n+1} with hub at x. Let u_i be adjacent to $w_i, 1 \leq i \leq n$ respectively. The graph obtained is denoted by $SW(r), n5$.

The number of vertices in $SW(r)$ is $n(r+2)+1$ and the number of edges is $2n(r+1)$. The diameter of $WW(r)$ is 6.

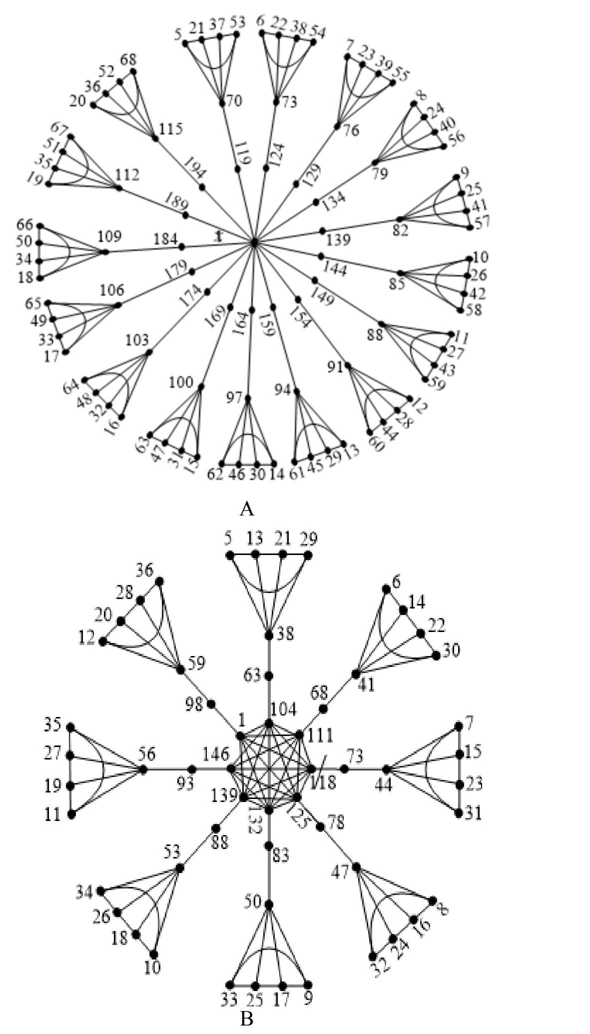

Figure 2 Radio labellings of SW(4) and KDW(4) which attains the lower bound.

***Theorem* 3.2**: The radio number of $SW(r)$ is given by $rn(SW(r)) = n(r+8)+2, n > 5$.

***Proof*:** Let $V(C_r^i) = \{v_{r(i-1)+j}: j = 1,2\ldots r\}, i = 1,2\ldots n$. We assume that the vertices v_i, u_i and w_i appear in the clockwise sense. Define a mapping $f: V(SW(r)) \rightarrow N$ as follows:

For $j = 1,2\ldots r$, $i = 1,2\ldots n$.

$$f\left(v_{r(i-1)+j}\right) = n(j-1)+i+4, f(w_i) = nr+3i+3$$

$f(u_i) = n(r+3)+5i+2, f(x) = 1$. See Figure 2 (a).

As the proof is similar to the theorem 3.1, we omit the rest of the proof.

The graph KDW(r)

***Definition* 3.3:** Let $x_i, 1 \leq i \leq n$ be the vertices of the star graph K_n. Let x_i be adjacent to $w_i, 1 \leq i \leq n$. Subdivide each edge $x_i w_i$ by $u_i, 1 \leq i \leq n$. Let W_i be a wheel with hub w_i. The graph obtained is denoted by $KDW(r), n > 6$.

The number of vertices in $KDW(r)$ is $n(r+3)$ and the number of edges is $2n(r+1)+\frac{n(n+1)}{2}$. The diameter of $KDW(r)$ is 7.

***Theorem* 3.3**: Let G be the graph $KDW(r)$. Then the radio number of G is given by

$$rn(G) = n(r+15)-6, n > 6.$$

***Proof*:** First we name the vertices of $KDW(r)$ in the same manner explained in Theorem 3.2. Define a mapping $f: V(KDW(r)) \to N$ as follows:

$f(v_{r(i-1)r+j}) = n(j-1) + i + 4, j = 1,2 \dots r,\ i = 1,2 \dots n.$

$f(w_i) = nr + 3i + 3, \quad i = 1,2 \dots n.$

$f(u_i) = n(r+3) + 5i + 2, \quad i = 1,2 \dots n.$

$f(x_i) = nr + 8n + 7i + 1., \quad i = 1,2 \dots n-1.$

$f(x_n) = 1.$ See Figure 2 (b).

As before we omit the details of the proof.

The graph $HW(r)$

***Definition* 3.4:** Let H_{n+1} be the helm with the pendant vertices $u_i, 1 \le i \le n$. Let $W_{r+1}^i = C_r^i + K_1$ be wheels with hubs $u_i, 1 \le i \le n$ respectively. The graph constructed is denoted by $HW(r), n > 11$.

The number of vertices and edges of $HW(r)$ are $n(r+2)+1$ and $2n(r+1)+n$ respectively. Its diameter is 6.

***Theorem* 3.4**: The radio number of $HW(r)$ is given by $rn(HW(r)) = nr + 8n + 2, n > 11$.

***Proof*:** Let $V(C_r^i) = \{v_{r(i-1)+j} : j = 1,2 \dots r\}, i = 1,2 \dots n$. Let $w_i, i = 1,2 \dots n$ and x be the vertices of the cycle and center of the inner wheel respectively. Also, we assume that the vertices v_i, u_i and w_i appear in the clockwise sense.

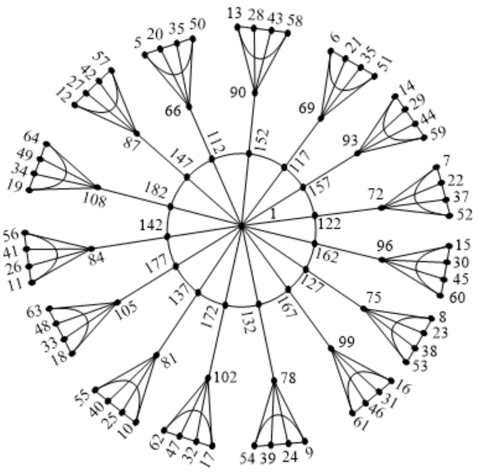

Figure 3 A graph HW(4) with n = 15 and a radio labeling which attains the bound.

Define a mapping $f: V(HW(r)) \to N$ as follows: $f(v_{r(i-1)r+j}) = n(j-1) + i + 4, j = 1,2 \dots r,\ i = 1,2 \dots n.$

$f(u_{2i-1}) = nr + 3(i+1), \quad i = 1,2 \dots \left\lceil \frac{n}{2} \right\rceil.$

$f(u_{2i}) = nr + 3\left(\left\lceil \frac{n}{2} \right\rceil + 1 + i\right), \quad i = 1,2 \dots \left\lfloor \frac{n}{2} \right\rfloor$

$f(w_{2i-1}) = n(r+3) + 5i + 2, \quad i = 1,2 \dots \left\lceil \frac{n}{2} \right\rceil.$

$f(w_{2i}) = n(r+3) + 5(\left\lceil \frac{n}{2} \right\rceil + i) + 2, \quad i = 1,2 \dots \left\lceil \frac{n}{2} \right\rceil.$

$f(x) = 1.$ See Figure 3.

The rest of the proof is left to the reader.

CONCLUSION

In this paper we completely determined the radio number of few graphs with extended wheels. The radio number problems for Honeycomb mesh, Oxide network, Silicate network, Butterfly networks…, are under investigation.

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