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Research Article

APPLICATION OF MAHGOUB TRANSFORM IN PARABOLIC BOUNDARY VALUE PROBLEMS

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A R T I C L E I N F O

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A B S T R A C T

In this paper, we apply Double Mahgoub Transform to solve the parabolic boundary value problem i.e. one dimensional Heat equation.

INTRODUCTION

The heat equation is parabolic boundary value problem, that describes the distribution of heat in a given region over certain time. Heat is the energy transferred from one point to another point. Heat flows from the point of higher temperature to one of lower temperature . For example the boundary value problem that govern's the heat flow in a rod. The heat conduction equation may have numerous solutions unless a set of initial and boundary conditions are prescribed. The boundary conditions are mainly of three types, which are as follows

Dirichlet condition: The temperature is prescribed all over the boundary surface. That is the temperature T(r, t) is function of both position and time. The special case T(r, t) = 0 on the surface of the boundary is called as homogeneous Dirichlet boundary condition.

Neumann condition: The flux of the heat , i.e. the normal derivative of the temperature $\frac{\partial T}{\partial n}$, is prescribed on the surface of the boundary . It may be function of both position and time. i.e.$\frac{\partial T}{\partial n}$ = f(r; t). sometimes, the normal derivative of temperature may be a function of position only or a function of time only. A special case $\frac{\partial T}{\partial n} = 0$ is homogeneous Neumann boundary condition. It is also called as insulated boundary condition which states that heat flow is zero.

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Robin's condition: A linear combination of the temperature and its normal derivative is prescribed on the boundary.

$$k\frac{\partial T}{\partial n} + hT = G\,(r\,,t)$$

where k and h are constants. It means that the boundary surface dissipates heat by convection. The homogeneous Robin's boundary condition is

$$k\frac{\partial T}{\partial n} + hT = 0$$

This means that heat is convected by dissipation from the boundary surface into a surrounding maintained at zero temperature.

The other boundary conditions such as heat transfer due to radiation obeying the fourth power temperature law and those associated with change of phase, like melting, ablation, etc. give rise to nonliner boundary conditions.

The PDE can be formally shown to satisfy

$$u_t = k\,u_{xx}\ ; 0 < x < L\,; t > 0$$

where u = u(x, t) represents the temperature of the rod at the position x at time t and k is thermal diffusivity of the material that measures the rod ability to heat conduction.

A lot of problems have been solved by integral transform such as Laplace [1], Fourier, Mellin, and sumudu [2], [3], Elzaki and Aboodh [5], Mahgoub [6]. Mozamel Omer Eshag [7] used double Laplace transform and double Sumudu transform to solve boundary value problems. K.Thangavelu, M.Pradeep and

K.Vinothini [8] used double Mahgoub transform to obtain solution of telegraph equation. Hchen and Ho [4] used two dimensional differential transform method for solving partial differential equations. In this paper we solve one dimensional heat equations by using double Mahgoub transform.

Mahgoub Transform

Definition: Let function f(t) defined for t ≥ 0 then Mahgoub transform of f(t) is the function H defined as follows:

$$M[f(t)] = v\int_0^\infty f(t)e^{-vt}\,dt \quad ; \quad t \ge 0$$

Double Mahgoub Transform

Definition: Let f(x; t);where $x, t \in R^+$ be a function, which can be expressed as a convergent infinite series then, it's double Mahgoub transform given by:

$$M_2\,[f(x,t);u;v] = H(u,v) = uv\int_0^1\int_0^1 f(x,t)e^{ux+vy}\,dx\,dt\,;\; x,t \ge 0$$

where, u, v are complex values.

Theorem

Double Mahgoub transform of first and second order partial derivatives are in the form:

1. $M_2\left(\frac{\partial f}{\partial x}\right) = u\,H(u,v) - u\,H(0,v)$
2. $M_2\left(\frac{\partial^2 f}{\partial x^2}\right) = u^2H(u,v) - u^2H(0,v) - u\,H_x(0,v)$
3. $M_2\left(\frac{\partial f}{\partial t}\right) = v\,H(u,v) - v\,H(u,0)$
4. $M_2\left(\frac{\partial^2 f}{\partial t^2}\right) = v^2H(u,v) - v^2H(u,0) - v\,H_t(u,0)$
5. $M_2\left(\frac{\partial^2 f}{\partial x\partial t}\right) = uv\,f(0,0) - uv\,H(u,0) + uv\,H(u,v) - uv\,H(0,v)$

Applications

In this section we establish the validity of the double Mahgoub transform by applying it to the heat equations. To solve partial differential equations by double Mahgoub transform, we use the following steps.

1. Take the double Mahgoub transform of partial differential equations.
2. Take the single Mahgoub transform of the conditions.
3. Substitute (ii) in (i)and solve the algebraic equation.
4. Take the double inverse of Mahgoub transform to get the solution Here we need the main

Equation

$$M_2(e^{ax+bt}) = \frac{uv}{(u-a)(v-b)}$$

Example:1 Consider the heat equation

$$u_t = u_{xx}\;;\; t > 0 \qquad (2.1)$$

with conditions

u(0; t) = 0; u(x; 0) = sin(x); $u_x(0,t) = e^{-t}$ (2.2)

Solution: Apply double mahgoub transform on equation (2.1)

$$M_2(u_t) = M_2(u_{xx})$$

$$v\,H(u,v) - v\,H(u,0) = u^2H(u,v) - u^2H(0,v) - u\,H_x(0,v)$$

$$\therefore (v - u^2)\,H(u,v) = v\,H(u,0) - u^2H(0,v) - u\,H_x(0,v) \qquad (2.3)$$

Apply single mahgoub transform on equation (2.2)

$$H(0,v) = 0\,;H(u,0) = \frac{u}{u^2+1}\;;\;H_x(0,v) = \frac{v}{v+1} \qquad (2.4)$$

putting in equation (2.3) we get,

$$(v-u^2)\,H(u,v) = v\left(\frac{u}{u^2+1}\right) - u^2(0) - u\left(\frac{v}{v+1}\right) = \frac{uv}{u^2+1} - \frac{uv}{v+1}$$

$$\therefore H(u,v) = \left(\frac{u}{u^2+1}\right)\left(\frac{v}{v+1}\right) \qquad (2.5)$$

taking double Mahgoub inverse

$$M_2^{-1}H(u,v) = M_2^{-1}\left(\frac{u}{u^2+1}\right)M_2^{-1}\left(\frac{v}{v+1}\right)$$

$$\therefore \mathrm{u(x;\ t)} = (\sin x)e^{-t}$$

Example:2 Consider heat equation

$$u_t = u_{xx} + \sin x\;;\quad t > 0 \qquad (2.6)$$

With the conditions,

$$u(0,t) = e^{-t}\,;u(x,0) = \cos x\;;\;u_x(0,t) = 1 - e^{-t} \qquad (2.7)$$

Solution: Apply double Mahgoub transform on equation (2.6)

$$M_2(u_t) = M_2(u_{xx}) + M_2(\sin x)$$

$$\therefore v\,H(u,v) - v\,H(u,0) = u^2\,H(u,v) - u^2H(0,v) - u\,H_x(0,v) + \frac{u}{u^2+1}$$

$$\therefore (v-u^2)\,H(u,v) = v\,H(u,0) - u^2H(0,v) - u\,H_x(0,v) + \frac{u}{u^2+1} \qquad (2.8)$$

Apply single mahgoub transform on equation (2.7)

$$H(0,v) = \frac{v}{v+1}\,;H(u,0) = \frac{u^2}{u^2+1}\;;\;H_x(0,v) = \frac{1}{v+1} \qquad (2.9)$$

putting in equation (2.8)

$$(v-u^2)\,H(u,v) = v\left(\frac{u^2}{u^2+1}\right) - u^2\left(\frac{v}{v+1}\right) - u\left(\frac{1}{v+1}\right) + \frac{u}{u^2+1}$$

$$(v-u^2)\,H(u,v) = u^2v\left(\frac{v-u^2}{(u^2+1)(v+1)}\right) + u\left(\frac{v+1-1-u^2}{(u^2+1)(v+1)}\right) \qquad (2.10)$$

$$\therefore H(u,v) = \left(\frac{u^2}{u^2+1}\right)\left(\frac{v}{v+1}\right) + \frac{u}{u^2+1}\left(\frac{v}{v+1}\right)\frac{1}{v}$$

Using partial fraction

Consider $\left(\frac{1}{v+1}\right)\frac{1}{v} = \frac{A}{v} + \frac{B}{v+1}$

Solving A = 1 , B = - 1

$$\therefore \left(\frac{1}{v+1}\right)\frac{1}{v} = \frac{1}{v} - \frac{1}{v+1}$$

$$\therefore H(u,v) = \left(\frac{u^2}{u^2+1}\right)\left(\frac{v}{v+1}\right) + \frac{u}{u^2+1}\left(1 - \frac{v}{v+1}\right) \quad (2.11)$$

Taking double Mahgoub inverse

$$M_2^{-1} H(u,v) = M_2^{-1}\left(\frac{u^2}{u^2+1}\right) M_2^{-1}\left(\frac{v}{v+1}\right) + M_2^{-1}\left(\frac{u}{u^2+1}\right)\left(M_2^{-1} 1 - M_2^{-1}\frac{v}{v+1}\right) \quad (2.12)$$

$$u(x,t) = \cos x \; e^{-t} + \sin x\,(1 - e^{-t})$$

is the required solution.

Example: 3 Consider heat equation,

$$u_t = u_{xx} - 3u + 3 \;;\; t > 0 \quad (2.13)$$

with the conditions

$$u(0,t) = 1 \;;\; u(x,0) = 1 + \sin x \;;\; u_x(0,t) = e^{-4t} \quad (2.14)$$

Solution: Applying double Mahgoub transform on equation (2.13)

$$M_2(u_t) = M_2(u_{xx}) - 3M_2(u) + M_2(3)$$

$$\therefore (v - u^2 + 3)H(u,v) = v\,H(u,0) - u^2 H(0,v) - u\,H_x(0,v) + 3 \quad (2.15)$$

Apply single Mahgoub transform on equation (2.14)

$$H(0,v) = 1 : H(u,0) = 1 + \frac{u}{u^2+1} \;;\; H_x(0,v) = \frac{v}{v+4} \quad (2.16)$$

putting in equation (2.15)

$$H(u,v) = \left(\frac{u}{u^2+1}\right)\left(\frac{v}{v+4}\right) + 1 \quad (2.17)$$

Taking double Mahgaub inverse,we get

$$u(x,t) = \sin x \,.\, e^{-4t} + 1$$

CONCLUSION

In this work, Double Mahgoub transform is applied to obtain the solution of heat equation of one dimensional, it may be conclude that double Mahgoub transform is very powerful and efficient for finding the analytical solution for a wide class of partial differential equation

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