



MODELLING THE SUCCESS OF FACEBOOK USING GRAPH THEORY

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ABSTRACT

In today's world, most of us use social media regularly. The prevalence of Social Media is increasing day by day. The most used online social networking (OSN) platforms are Facebook, Instagram, Twitter, WhatsApp. Graph Theory, in mathematics is known for its many applications in real life. One such is modelling social structures based on different kinds of relationships between people or groups of people. As the new age social relationships of groups of people reflect in the social networking media that are extensively used, Facebook, Instagram, Twitter and WhatsApp can be modelled using Graphs. Facebook, in itself owns both WhatsApp and Instagram. The massive success of Facebook is amusing to most competitors in the business. In this paper, it is discussed, through concepts of Graph Theory how the interconnection between different social media owned by Facebook works and it is tried to explain the success of Facebook.

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INTRODUCTION

Social networks and the techniques to analyse them existed since decades [1]. A Social network is a network of relationships or interactions, where the nodes consist of people or actor, and the edges consist of the relationships or interactions between these actors [2]. There can be several types of social networks like email network, telephone network, collaboration network. Recently online social networks like Facebook, Twitter, Linked In etc. have been developed and gained massive popularity within very short amount of time and gathered large number of users. Facebook is said to have more than 500 million users in 2010 [3]. The field of social networks and their analysis has evolved from graph theory, statistics and sociology and it is used in several other fields like information science, business application, communication, economy etc.

The online social network graph may be very large. It is supposed to contain millions of nodes and edges. Social networks are dynamic i.e. they are continuously expanding and have new evolutions. There are small and large communities within the social graph and graph theory offers several concepts which can be used to understand and study the social groups.

Graph Theory

Graph Theory is a well-known area of discrete mathematics which deals with the study of graphs. A graph is a mathematical structure that is used to model the pairwise

relations between objects. There are many theoretical developments and applications of Graph Theory, not only to different branches of mathematics, but also to various other fields of basic sciences, technology, social sciences, computer science etc. They are also widely used as efficient tools to model many practical and real-world problems in physical, biological, social and information systems.

One of the increasing use of Graphs is to model social structures based on different kinds of relationships between people or groups of people (Kenneth H. Rosen, 2012). Here, individuals or organizations are represented by vertices, relationships between individuals or organizations are represented by edges of graphs. The study of social networks, especially online social networking (OSN) have become rampant in multidisciplinary research area with the increasing use of social media.

Social Network Analysis (SNA)

The goal of SNA is to identify who the key actors are and what positions and actions they are likely to take in networks of individuals. It has been applied to networks of individuals (Krackhardt, 1996) as well as networks of organizations (Brennan, 1999). In SNA, interrelations and connections are represented as networks where the nodes are either individuals or organizations with arcs representing associations (Krackhardt, 1996). The arcs may be directed or undirected; undirected arcs indicate a mutual relationship.

Social Media

Social media has become an integral part of communication with people all over the world. It is not only used just by young adults but people from all age group now, as opposed to

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that over the last decade. Many find it a blessing and they love the connectivity. Nowadays, more than physical meet, being active on social media networks like Facebook, WhatsApp etc. has become more necessary.

In earlier times, the communication facilities were poor, where people were not able to get connected to their friends and relatives often. The cost of communication was also high. Making a simple phone call was also a mammoth task.

But, today, it has all changed. Within seconds, we are able to connect with the help of the technological advances. Not only, to one person, but, within fraction of seconds, we can connect to many recipients at one single time. All the credit goes to social media services.

Facebook, WhatsApp and Instagram

Facebook is an application that can be used on the web or downloaded as a mobile application. Users require to sign in using their e-mail ids or phone numbers. They need to share some information about themselves, like name, date of birth, relationship status, etc. However, one can change the settings in a way that the visibility of these information is controlled. Facebook works in the mechanism of adding friends by sending friend request to any other existing user. If the request is accepted, two people become Facebook friends and can chat, share status, comment on each other’s posts or even tag each other. There are Facebook groups as well.

WhatsApp is an application, which can be downloaded, either on mobile or on the desktop. After that, it can be used by signing up with phone number. It is user-friendly. It instantly sends messages to anyone and anywhere in the world. There are no restrictions on the number of messages or pictures shared. WhatsApp ensures privacy settings like hiding the profile picture, hiding the status, etc. It also gives the option to check whether the recipient has viewed the message or not. Users can also be blocked if one wants that restricts the blocked user from sending any message or viewing the other person online.

Both Facebook and WhatsApp take less time for communication between known and unknow groups of people provided there is internet connectivity. They tend to have sound privacy system. They have online calling and video calling facility and are easy to use. Even a novice mobile user can use these applications.

Instagram is an application which is popular because of its unique style of sharing pictures with online followers. One has to sign up using e-mail id or phone number. Then the user can follow other people on it and get to see their posts. Posts are essentially pictures with captions as updated by the users. It has increasingly become a trend among the youth to see pictures on Instagram now-a-days.

Facebook in itself, has a user base of over 2.3 billion users now. Facebook also owns the two other most prominent social media sites, Instagram and WhatsApp. Facebook is a clear winner in this business for a unique reason, that is its user network. In this paper, it is described how different concepts of Graph Theory is related to social media platform. Also, using these concepts, an explanation of the massive success of Facebook is given.

THEORY

Graph Theory Terminology

Graph: A graph G is denoted as (V, E), where :

1. $V = \{v_1, v_2, v_3, \dots\}$ is called the vertex set of G and the elements of V are called the vertices (or points or nodes);
2. $E = \{e_1, e_2, e_3, \dots\}$ is called the edge set of G and the elements of E are called edges (or lines or arcs).

Commonly, the graph is denoted as $G = (V, E)$. The vertex set and edge set of a graph G are also written as $V(G)$ and $E(G)$ respectively.

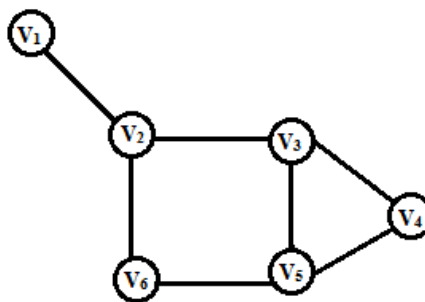


Figure 1 An example of a graph

The graph given in Figure-1 has vertex set $V = \{V_1, V_2, V_3, V_4, V_5, V_6\}$ and edge set $E = \{(V_1, V_2), (V_2, V_3), (V_2, V_6), (V_3, V_5), (V_3, V_4), (V_4, V_5), (V_5, V_6)\}$.

Edge: An edge is a line at which vertices are connected in the graph. Edges are denoted by $E = (U, V)$ it is a pair of two vertices.

Order and Size of a Graph: The order of a graph G, is the number of its vertices and the size of G, is the number of its edges. A graph with p-vertices and q-edges is called a (p,q)-graph.

Trivial Graph: The (1, 0)-graph is called a trivial graph. That is, a trivial graph is a graph with a single vertex.

Null Graph: A graph without edges is called an empty graph or a null graph.

Degree of a vertex: The number of edges incident on a vertex v, with self-loops counted twice, is called the degree of the vertex v and is denoted by $deg(v)$ or simply $d(v)$.

Isolated vertex: A vertex having no incident edge is called an isolated vertex. In other words, isolated vertices are those with zero degree.

Pendant vertex: A vertex of degree 1, is called a pendent vertex or an end vertex.

Disconnected graph: A graph G is said to be disconnected if there exist two vertices in G such that no edge in G has those vertices as endpoints.

Directed graph: A directed graph in which each edge is represented by an ordered pair of two vertices, e.g. (V denotes an edge from V_i to V_j (from the first vertex to the second vertex). Every edge has a specific direction.

Undirected Graph: An undirected graph contains an unordered pair of vertices.

Complete Graphs: A simple graph $G = (V, E)$ with n mutually adjacent vertices is called a complete graph G and it is denoted by K_n or a simple graph $G = (V, E)$ in which every vertex is mutually adjacent to all other vertices is called a complete graph G .

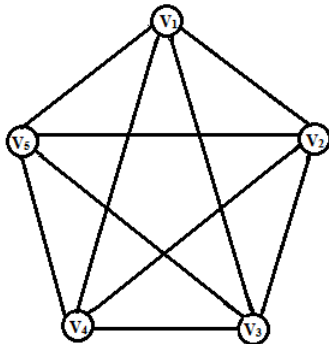


Figure 2 An example of a Complete Graph

Regular graph: In a graph if all vertices have same degree (incident edges) k then it is called a regular graph.

Walks: A walk in a graph G is an alternating sequence of vertices and connecting edges in G . In other words, a walk is any route through a graph from vertex to vertex along edges.

Closed Walk: If the starting and end vertices of a walk are the same, then such a trail is called a closed walk. A walk can end on the same vertex on which it began or on a different vertex. A walk can travel over any edge and any vertex any number of times.

Trail: A trail is a walk that does not pass over the same edge twice. A trail might visit the same vertex twice, but only if it comes and goes from a different edge each time.

Tour: A tour or a closed trail is a trail that begins and ends on the same vertex.

Path: A path is a walk that does not include any vertex twice, except that its first vertex might be the same as its last.

Cycle: A cycle or a circuit is a path that begins and ends on the same vertex.

Cycle Graph: A cycle graph or circular graph is a graph that consists of a single cycle, or in other words, some number of vertices connected in a closed cycle.

Triadic Closure

If two people in a social network have a friend in common, then there is an increased likelihood that they will become friends themselves at some point in the future. This principle is called Triadic Closure.

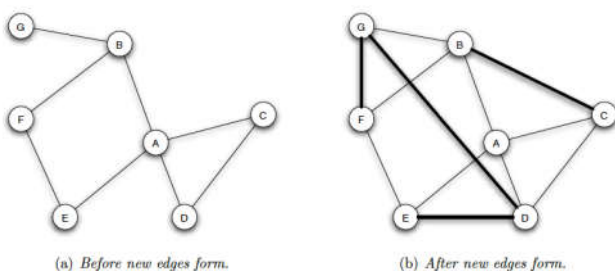


Figure 3 Triadic Closure

As shown in the figure, if nodes B and C have a friend (node) A in common, then the formation of an edge between B and C produces a situation in which all three nodes A, B, and C have edges connecting each other—a structure which can be referred to as a triangle in the network. The term “triadic closure” comes from the fact that the B-C edge has the effect of “closing” the third side of this triangle.

If a social network is observed at two distinct points in time, then in the second observation, it is generally found that a significant number of new edges that have formed through this triangle-closing operation, between two people who had a common neighbour in the previous observation. Thus, triadic closure principle holds true for networks of people.

The Strong Triadic Closure Property is that if a node has strong ties to two neighbours, then these neighbours must have at least a weak tie between them, which is a fundamental rule for networks of people.

DISCUSSION

Facebook and Graph Theory

Facebook is a social media platform, where one has to sign up using e-mail id or phone number to become a user. They also have to fill up some basic information about their whereabouts. Once a user is created, it becomes a **Vertex**. He can become friends with other users on it by sending a friend request. The other user can accept or reject it. If they accept it, both of them get connected, as a graph which has two vertex and one edge. Both of them can share their posts through text or images and get to see their posts. Similarly, people who will follow him back, will be able to see his posts.

The number of friends of a user is the **Degree** of the vertex of that user.

Also, two vertices which are incident with a common edge, i.e. a connection of friendship in case of facebook, are said to be **Adjacent**. So, all the friends of the person considered here are **Adjacent**, in terms of Graph Theory.

Let’s consider three users A, B and C on Facebook.

When A, B and C individually joins Facebook, they are encouraged to build network. They can discover new profiles very easily. Even Facebook has this option of ‘Finding Friends’ and Friend Suggestions which increases the visibility of related profile. The triadic closure plays in this way: If A and B are friends and B and C are friends on Facebook, there is a high chance that A and C will also become friends. A will see B’s friend C in the friend suggestions and thus the chances of A and C becoming friends on Facebook increases. When A and C becomes friends, A’s another friend D will see C as his/her suggestion. This is the power of triadic closure in creating networks in Facebook.

WhatsApp and Graph Theory

Communication needs both sender and receiver. In WhatsApp there can four types of communication, a) one to one b) a single sender with many receivers c) many sender single receiver and d) many senders many receivers.

In WhatsApp when a user first signs up, he becomes a **node/vertex** of a graph. Similarly, there are few contacts of his, who also individually act as **nodes**. In this scenario, where

no communication has taken place among these contacts, it is a **Null Graph**. Each node in this case are **Isolated Vertices**.

In this example, if one user has five contacts, so this graph becomes a null graph with six nodes.



Figure 4 When all the people are idle with no communication between them.

Now, if the new user starts messaging one of his/her contact, they are connected. This connection can be represented as an edge between the two nodes.

This association is represented as the following, where both the vertices are **Pendant Vertex**, with only one degree. And the resultant graph consisting of these two individuals is a **Connected Graph**. It is illustrated in Figure 5.

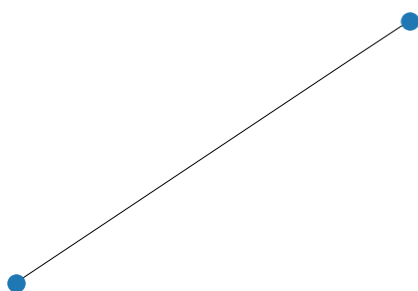


Figure 5

However, the overall graph, of six individuals, where only two of them are connected online through WhatsApp, the overall graph is a **Disconnected Graph** as shown in Figure 6.



Figure 6

In WhatsApp, groups can be formed, where many users need to interact with many other users at the same time. For example, if six users need to contact with each other, the situation is like a

Complete Graph. This **Complete Graph** will be on six edges and all the pairs of vertices will be connected to one another by an edge signifying they are interacting.

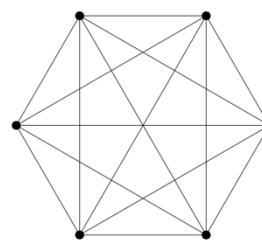


Figure 7 Complete Graph with six vertices.

In the ideal situation of a group chat, each person is interacting with all the others in the group chat. So, in a group chat of six people, each person is interacting with five other users.

So, the **Degree** of each vertices is 5, where one vertex represents a user. Similarly, in a group chat of five people, each user is interacting with four other users, which implies that the Degree of each vertex in this case is 4.

This means, that the complete graph in Figure 7 is also a **Regular Graph** in this scenario of a WhatsApp group chat.

In WhatsApp, there can be situation such that one message, such as news is circulated among a group of users among whom all of them maybe related or unrelated to one another.

Explaining further, suppose, user A sends a message to B, and B sends to users C, D and E. E happens to know user A and sends the message to users A and F. So, there exists a **Walk** between A to F, through B and E.

Also, the message from A to B to E and then back to A is a **Cycle Graph** and it is a unique **closed Path**. This can also be considered as a **Directed Graph** as the messages are directed from A to B, then B to E and finally E to A.

Instagram and Graph Theory

Instagram is a social media platform, where one has to sign up using e-mail id or phone number to become a user. Once a user is created, it becomes a vertex. He can follow other people on it and get to see their posts. Similarly, people who will follow him back, will be able to see his posts. But it can be certainly assumed that there are various other users of Instagram as well. So, as long as he does not follow anyone and no one follow him, it remains a **Disconnected Graph**.

The number of followers of a user is the Degree of the vertex of that user.

Also, two vertices which are incident with a common edge, i.e. a connection of following in case of Instagram, are said to be **Adjacent**. So, all the users following the person considered here are **Adjacent**, in terms of Graph Theory. Similarly, all the people whom the person is following are **Adjacent** to him.

In Instagram, like that of Facebook, Triadic Closure comes into play.

When A, B and C individually joins Instagram, they are encouraged to build network and increase their activities. They can discover new profiles very easily. Each of A's followers will discover B prominently is A and B are related to each other. Similar algorithms of Instagram work for C and his/her followers. This results in increased network building among the users of Instagram.

Success of Facebook

Facebook owns both Instagram and WhatsApp. Considering triadic closure principle which works in Facebook and Instagram, and how they are encouraging users on building their networks explain the tremendous success of Facebook, as compared to any other competitor like Twitter, can be easily explained.

CONCLUSION

In this work, the various relationships of social media platforms such as Facebook, WhatsApp and Instagram have been modelled with the concepts of graph theory.

The focus of this research concentrated on trying to explain the success of Facebook using the triadic closure principle. This extends the scope of graph theory and study of networks to the social networking platforms to model and take the studies further.

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